heat transfer in the range of small and medium values of the $\kappa x/\bar{\delta}^2$ parameter under the assumption that the wall zone has been neglected.

NOTATION

a, thermal diffusivity in the granular bed; L, plate length; h, wall zone thickness; q, local thermal flux; $\bar{q}$, thermal flux averaged along the plate; r, latent heat of phase transition; R, grain radius; T, T_w, and T_s, temperature, plate temperature, and temperature at the film's outer boundary, respectively; U, filtration rate; x and y, longitudinal and transverse coordinates, respectively; α, heat transfer coefficient; δ and $\bar{\delta}$, local and mean film thickness, respectively; λ_L, λ_w, and λ_e, thermal conductivity of the liquid, the wall zone, and the main body of the film, respectively; ρ_L and ρ_g, densities of the liquid phase and vapor, respectively; Π, permeability of the bed; φ, angle between the longitudinal coordinate x and the gravity acceleration vector g; ω, angular velocity; μ and ν, dynamic and kinematic viscosity of the liquid, respectively.

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PROPAGATION OF VIBRATIONS IN A SUSPENDED GRANULAR BED

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The influence of the elasticity and relative motion of the continuous medium on the hydrodynamics of a suspended vibrating bed is discussed. A solution is given for the boundary-value problem of small pressure disturbances propagating in the bed. The results are compared with experimental data and calculations based on existing models.

1. PHYSICAL MODEL

The action of vibrations on disperse materials for the purpose of intensifying heat- and mass transfer processes has been utilized for some time now with optimistic results [1, 2]. On the other hand, the theory of vibrofluidization [3-5] is far from complete in either the quantitative or the qualitative aspect. It fails to describe high-frequency (>10 Hz) resonance effects, which have been noted by many researchers, including Kroll [3] and Gutman [4], and which enhance heat and mass transfer significantly at their peak development [2]. The discrepancy with experimental results is an outgrowth of a common practice in the mechanics of fluidized systems (FS's) [6], namely the representation of the gaseous medium as an incompressible fluid, which limits the application of the theory to parameters that support the customary relation between the equilibrium pressure P_0 and its variation p: $P_0 \gg p$. The latter corresponds quite well to "low" ($\Delta p_b \ll P_0$) fluidized beds, but

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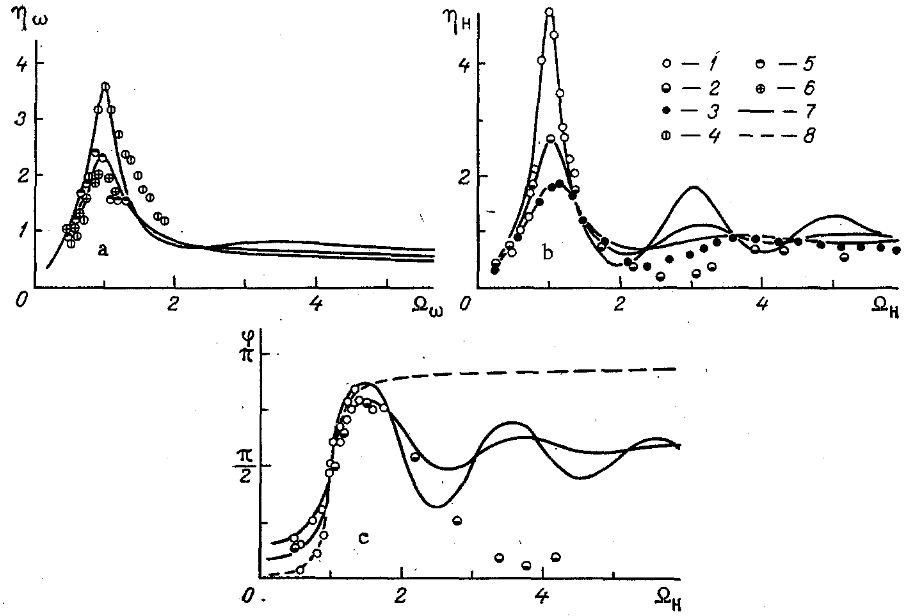

Fig. 1. Resonance curves (a, b) and phase response curves (c) of a vibrating fluidized bed. 1) f = 20 Hz, $\Theta_v = 0.26$; 2) 24, 0.51; 3) 77, 0.75; 4) $H_0 = 0.12$ m, $\omega_0\tau_V = 0.379$; 5) 0.07, 0.6; 6) 0.05, 0.718; 7) calculated according to Eq. (11); 8) according to Eq. (16), corundum; 2) from [43]; 3, 4) from [32]. φ, rad.

fails in application to FS's in which the pressure differences are close to the equilibrium value. Examples of such systems are moving high (>5-10 m) fluidized beds [6], vacuum fluidized beds [7], and vibrating fluidized beds. When the compressibility of the gas is taken into account in the latter case, the resonance parameters, dynamic elasticity, and swelling of the packed particulate material can be adequately estimated [8-10].

The effects of elasticity of the gas in FS's were first brought to attention by Davidson [11], who drew an obvious analogy with the bubbling of a gas in a liquid. Binie [12] elaborated these notions, imparting elasticity to the entire mass of the fluidizing agent, and demonstrated the practicality of this approach for particles smaller than 0.2 mm in diameter. The hypothesis of a concentrated (in bubbles, in the subgrid chamber, in the space beneath the bed, etc.) elasticity [11] has been developed in later studies aimed at coarse granular materials [3, 13-18]. However, neither the frequency range nor the behavior of the dependences in the proposed models was modified in comparison with [6], whereas in experiments with finely dispersed FS's the same authors noted laws that are not inherent in the given models [3, 4, 16, 17].

Referring to the general physical model of the FS as a two-fluid two-velocity continuum [6] and, in contrast with [19], retaining the compressibility of the continuous phase, we write

$$D_t(\varepsilon\rho) + \operatorname{div}(\varepsilon\rho\mathbf{U}) = 0, \quad D_t\varepsilon - \operatorname{div}((1+\varepsilon)\mathbf{V}) = 0; \tag{1}$$

$$\rho D\mathbf{U} + \operatorname{grad} P + \beta(\mathbf{U} - \mathbf{V}) = 0, \quad \rho_b D\mathbf{V} - \beta(\mathbf{U} - \mathbf{V}) - \rho_b \mathbf{g} = 0. \tag{2}$$

The x axis is directed vertically downward.

2. ANALYSIS OF THE MOTION OF THE CONTINUOUS PHASE

Estimating the significance of the inertial terms in Eq. (2), we obtain

$$\mathbf{V} \sim V_v; \quad D_t\mathbf{V} \sim V_v\omega; \quad (\mathbf{V}\nabla)\mathbf{V} \sim V_v^2/H, \; D\mathbf{V} \sim A_v\omega^2\left(1 + \frac{A_v}{H}\right) \approx A_v\omega^2;$$

$$\mathbf{U} \sim (1+\Theta_v)V_v; \quad D_t\mathbf{U} \sim \mathbf{U}\omega; \quad (\mathbf{U}\nabla)\mathbf{U} \sim \mathbf{U}^2/d. \tag{3}$$

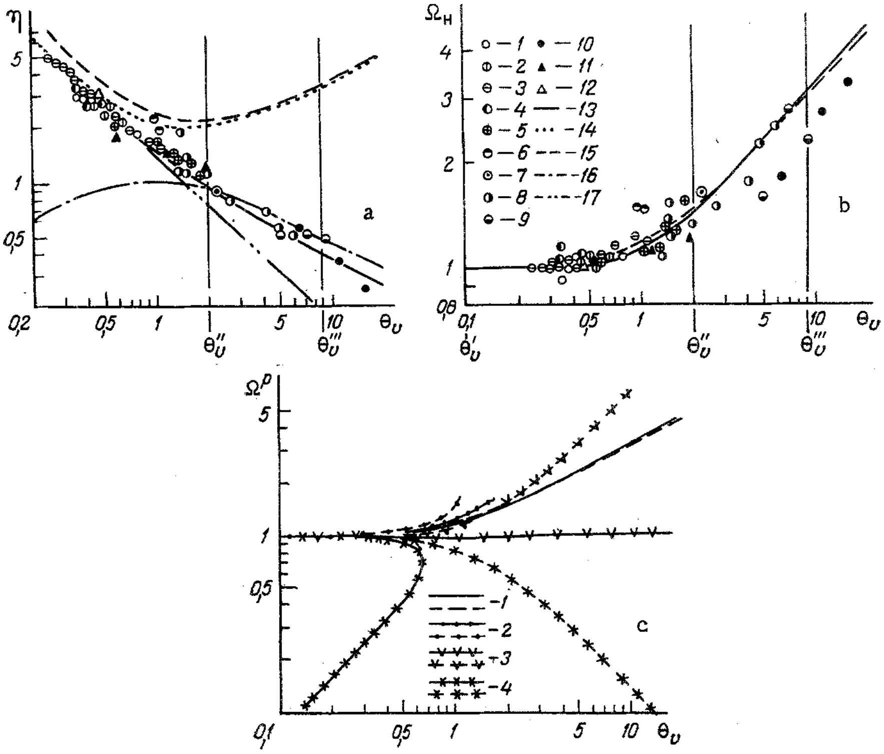

Fig. 2. Resonance pressures (a) and frequencies (b, c) in a vibrating fluidized bed. a, b: 1) d = 45 μm; 2) 57; 3) 72; 4) 90; 5) 140; 6) 180; 7) 220; 8) 280; 9) 357; 10) 580; 13) calculated according to (11); 14) according to (12); 15) according to (16); 16) according to (13); 17) η_ω (11), f = 10-100 Hz, V_V = 0.25-0.45 m/sec; 1-10) corundum; 11) sand; 12) glass; 1, 2, 7-10) from [32]; 3, 5) [27]; 4, 6) [29]; 11) [3]; 12) [4]. c: 1) var H ($\Omega_H = \Omega_H{}^P$); 2) var ω, const A_V ($\Omega_\omega < \Omega_\omega{}^P$); 3) var ω, const V_V ($\Omega_\omega = \Omega_\omega{}^P$); 4) var ω, const K_V ($\Omega_\omega > \Omega_\omega{}^P$). Solid curves: calculated according to (11); dashed curves: calculated according to (16).

Allowance is made here for the fact that the vibrational particle velocity varies over distances of the order of the bed height H, as opposed to a disperse system, in which this scale is only ~d, and convective accelerations compete with the local accelerations. Consequently, a rigorous analytical solution of Eqs. (2) is possible only in quasisteady motion of the gas [20], when the inertial forces* in it are considerably smaller than in the dispersion medium. Introducing the rate and depth of slippage of the phases, along with the local (vibrational) and convective Reynolds numbers [20], we obtain simple relations for assessing the hydrodynamics of the suspended vibrating bed:

$$|\mathbf{U}-\mathbf{V}| = \Theta_v V_V;\quad \frac{S}{d} \sim \frac{A_V}{d}\,\Theta_v;\quad \frac{F}{F_b} = \frac{\rho D U}{\rho_b D V} \sim \frac{\rho}{\rho_b}\left(1+(1+\Theta_v^2)\,\frac{A_V}{d}\right);$$

$$\frac{R_c}{R_V} \sim \left((\Theta_v V_V + U_0)\,\frac{d}{\nu}\right)/(\omega d^2/\nu) = \frac{S}{d} + U_0/(\omega d). \tag{4}$$

The fundamental governing parameter in Eq. (4) is the dimensionless vibration frequency Θ_v. The following relations hold for small values of $\Theta_v < \Theta_v' \sim$ 0.1-0.01 (when $A_V/d \sim 10^1$-10^2, $A_V/H < 10^{-2}$, and $\rho_b/\rho \sim 10^3$) in the absence of forced blowing ($U_0 = 0$):

$$|\mathbf{U}-\mathbf{V}|/V_V < 0.1-0.01;\quad S/d \lesssim 1;\quad F/F_b < 0.01;\quad R_c < R_V < 0.1;$$
$$d \lesssim d' \sim (150/V_V)\,(\rho/\rho_b)\,(1-\varepsilon)^2\,\nu/\varepsilon^3. \tag{5}$$

*The most significant of the other forces characterizing inertial effects in the fluid is the force F_m associated with the influence of the apparent mass. However, its contribution, which has been determined previously [21], is almost an order of magnitude smaller than F.

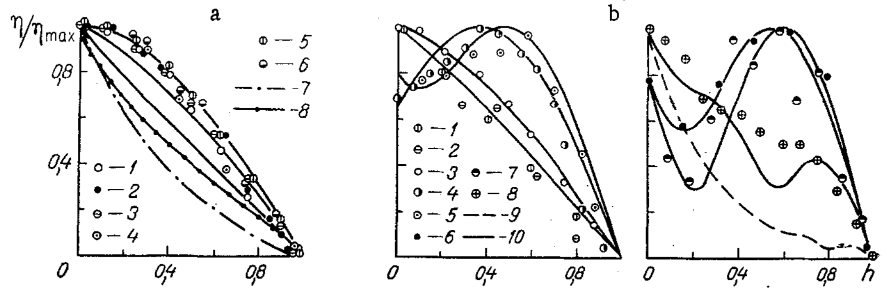

Fig. 3. Mode shapes of the gas-pressure vibrations in a vibrating fluidized bed. a) $\Omega \simeq 1$: 1) $\Theta_v = 0.359$; 2) 0.487; 3) 0.545; 4) 0.606; 5) 0.632; 6) 1.494; 7) 0.4; 8) 0.59; 1-6) from [32]; 7) calculated according to [3]; 8) [4]; the numbers refer to the curves calculated according to (11): 1) $\Theta_v = 0.1$; 2) 1; 3) 100. b) $1 - \Omega_H = 0.54$, $\Theta_v =$ 0.359; 2) 0.8, 2.465; 3) 1, 0.359; 4) 1.65, 0.245; 5) 2.25, 0.508; 6) 2.4, 0.366; 7) 2.53, 0.187; 8) 4.67, 0.517; 9) 10, 0.516; 1, 3, 4, 6, 7) from [32]; 2, 5, 8) [10]; 9, 10) calculated according to (11).

The dynamics of collective motions in the heterogeneous medium is practically independent of inertial effects in the gas here. The motion of the gas relative to the particles, in turn, is characterized as quasilaminar and quasisteady. This state produces dynamic consolidation and "blow-through resistance" on the part of the heterogeneous system, which vibrates essentially without expansion and slippage of the phases, much like a continuous medium. The bed closely resembles a stationary medium in terms of its heat- and mass-transfer properties. The excitation of relative motion in it requires that the parameters (V_v, U_0) can be varied in such a way as to "unfreeze" the gas and thus achieve the relation $S/d > 1$.*

In the frequency range $\Theta_v' - \Theta_v''$, where $\Theta_v'' \approx 2$, the ratio F/F_b increases to ~0.1, the contribution of the convective component increases, $R_c/R_v \sim 1\text{-}20$, and intralayer ($1 < S/d \ll H/d$) unsteady gas filtration develops in the granular interstices. On the one hand, the solid particles are capable of inertial motion relative to the gas ($S/d > 1$), and the granular material can loosen up progressively by layers from top to bottom.† On the other hand, the jet flows and filtration mixing will probably be accompanied by microcirculation currents between the particles along with their growth and transverse vibration. While these phenomena do not affect the nature of the collective vibrations in the bed, they are significant in other, more subtle processes associated with heat and mass transfer and with chemical reactions;‡ this assertion is confirmed indirectly by studies of the optimization and intensification of transport processes in a vibrating bed [26-28].

Phase slippage and convective flows develop in the given regime, preserving the predominantly viscous nature of the filtration process, since the dimensions of the interstices ($\lesssim d$) always remain smaller here than the effective depth δ_h of propagation of shear vibrations in the viscous fluid:

$$d \lesssim \delta_h \sim \sqrt{2\nu/\omega} = d'', \quad \text{or} \quad R_v < R_v'' \simeq 2. \tag{6}$$

According to (6), the limiting particle size d" at frequencies of 10-100 Hz is 0.7-0.2 mm, corresponding to the upper grain-size limit for effectively vibration-fluidized disperse materials [2, 29] with "uncharacteristic" attributes in their behavior [3, 4, 16, 17].

*This conclusion is highly consistent with fluidization practice for finely disperse materials [1, 2, 6, 23-25]. In view of the limited vibrational velocity capabilities of vibration engineering, better results are attained by means of auxiliary forced blowing (U_0) [26].

†This vibration-induced loosening of finely disperse powders is observed, e.g., in high-speed motion pictures of the process [25].

‡In particular, the occurrence of phase slippage S creates an active gas-exchange zone $\delta \simeq S$ between the core of the layer and the vibrating surface. The motion of the gas particles in the core, tending in general along curved paths, resembles the motion of solid grains in the well-known model of Zabrodskii [29] and accelerates (by an unsteady mechanism [30]) the diffusion processes in the treated product by intensifying mass transfer [28].

In the interval $\Theta_v''-\Theta_v'''$, where $\Theta_v'''\approx 9$, the flow structure changes, the depth of the viscous layer δ_h becomes smaller than the pore diameter (~d), and an inertial core is formed in the middle of the interstices. The inertial forces of the gas and the particles in Eq. (2) become commensurate in this case, and the path of the fluid particle S increases from ~10^1d to ~10^2d, attaining values of almost the same order of magnitude as the bed height H, and the phase slippage rate is equal to (2-9)v_V, or 10^0-10^1 m/sec.*

For $\Theta_v>\Theta_v'''$ we have $\delta \ll d$ and $S/H > 1$. As a result, the gas easily "slips through" from the space above the bed to the clearance below the bed and back again during one vibration period without any appreciable effect on the individual elements or on the packed bed as a whole. For the customary parameters ($K_V \leqslant 5$-10 [29, p. 204]) this bed is thrown up above the bottom like a rigid porous body, completely consistent with the theory [3]. It is therefore suitable to use the quasisteady-state equation of motion of the gas in (2) under the condition $\Theta_v\leqslant\Theta_v'''$.

3. PROPAGATION OF PLANE WAVES IN A BOUNDED BED

Owing to the high level of the dissipative forces [31], nonlinear effects are weak in a vibrating finely disperse packing, even when the deviations of the gas pressure from the equilibrium pressure are not too small [32]. This means that the system (1), (2) can be treated in the linear approximation. Invoking the small perturbation method: $P = P_0 + p$, $\varepsilon = \varepsilon + e$, $V = v$, and eliminating variables, we obtain an equation for the propagation of elastic isothermal pressure waves of an ideal gas in a suspended granular bed:

$$D_{tt}p - a_0^2(1+\tau_v D_t)\nabla^2 p = 0. \qquad (7)$$

Equation (7)† describes the situation when the excess pressure in the fluid cannot relax under deformation of the interstices of the FS and creates a bulk elastic resistance to motion. This resistance increases as the particle diameter is decreased ($\Theta_v \to 0$) and becomes much larger than the filtration frictional resistance. Considering the case of small ($L \ll \lambda$ with allowance for the constraints of the problem [29]) transverse dimensions of the bed, we arrive at the problem of the propagation of plane waves: $\nabla^2 = D_{xx}$.

Boundary Conditions. We choose the origin x = 0 on the surface of the layer, in which case the bottom of the apparatus corresponds to x = H. Taking the nonseparated nature of the motion into account, we express the velocity of the gas at the lower boundary in terms of the vibrational velocity of the bottom: $U(H, t) = \mathrm{Re}[iV_V \exp(i\omega t)]$. Unlike the customary approach used in the continuum description [10, 34, 35], the particles have a certain free motion. Consequently, for nonimpact [36] regimes we have $V(H, t) = U(H, t) + (D_x p)/\beta$. This relation enables us to describe the dynamics of the suspended layer within the framework of a unified boundary-value problem not only during simultaneous motion of the particles with the bottom ($\Theta_v \ll 1$), but also with periodic separations ($\Theta_v > 2$). For the case of strong vibrations ($A_V\omega^2 \gg g$) we obtain the condition at the lower boundary from Eq. (2):

$$D_x P(H, t) + \tau_v D_{xt} P(H, t) = \mathrm{Re}(j_V \exp(i\omega t)). \qquad (8)$$

The fluctuations of the gas pressure decay at the upper boundary. In the first approximation,‡ which is confirmed by practice [8, 39, 40], we can assume that

$$P(0, t) = 0. \qquad (9)$$

Obtaining a steady-state solution of Eqs. (7), (8), and (9) in the form $p = Q(x)\exp(i\omega t)$, we reduce the problem to the corresponding problem for the configurational part:

$$\ddot{Q}(x) + K^2 Q(x) = 0;\quad K^2 = \omega^2/(a_0^2(1+i\Theta_v));$$
$$Q(0) = 0;\quad Q_x(H) = j_V/(1+i\Theta_v). \qquad (10)$$

*The adequacy of this estimate is corroborated by direct experiment [22] and by tracer tests conducted by the authors in collaboration with M. A. Afanas'ev.

†Equations of this type are used extensively in the description of various kinds of relaxation processes [30]. Attempts to use the equations for the analysis of vibrofluidization have also been reported [10, 29, 33].

‡More rigorous approaches allowing for the finiteness of the vibrations of the upper boundary and momentum losses with the expelled gas have been described [10, 37]. A critique of the hypothesis [38] of internal wave reflection from the free boundary of the bed is also given in [37].

Seeking a solution of Eq. (10) in the form $Q(x) = B(\exp(\Lambda_1 x) + \exp(\Lambda_2 x))$ and taking the time-varying part into account, we obtain the required relation

$$p(x, t) = \mathrm{Re}(Q(x)\exp(i\omega t)) = |Q(x)|\cos(\omega t + \varphi);$$

$$|Q(x)| = \eta(x) P_V; \quad P_V = \rho_{co} V_V a_0;$$

$$\eta(x) = \sqrt{\mathrm{sh}^2(K_2 x) + \sin^2(K_1 x)} / \sqrt{r_v(\mathrm{sh}^2(K_2 H) + \cos^2(K_1 H))};$$

$$K_1 x = \pi \frac{\Omega x}{H} \sqrt{1 + r_v}/(2\sqrt{2r_v}); \quad K_2 x = -\pi \frac{\Omega x}{H} \Theta_v/(2\sqrt{2r_v(1 + r_v)});$$

$$\varphi = \sum_{n=1}^{4} \varphi_n; \quad \mathrm{tg}\,\varphi_1 = -\Theta_v; \quad \mathrm{tg}\,\varphi_2 = -(1 + r_v)/\Theta_v;$$

$$\mathrm{tg}\,\varphi_3 = -\mathrm{th}(-K_2 H)\,\mathrm{tg}(K_1 H); \quad \mathrm{tg}\,\varphi_4 = \mathrm{cth}(-K_2 x)\,\mathrm{tg}(K_1 x). \tag{11}$$

4. DISCUSSION

It follows from the solution that standing gas-pressure (and particle-velocity) waves are established in a suspended bed with periodicity characterized by the parameter $\Omega x/H$, where they modulate a complex system of standing and traveling waves [8, 41] and correlate weakly with the porosity waves [42], which are described by an equation of higher order than (7). The sharpness of the resonance peaks and the decay rate of the resonance pressure curves depend on the way in which the parameter Ω is varied (Fig. 1). The interaction forces between the phases diminish as the vibration frequency ω is increased. The pressure vibrations p_ω rapidly become aperiodic in this situation, even in a bed of fine particles (50-100 μm), and the higher-order resonances (and also the first resonances at frequencies ≥50 Hz) are practically nonexistent. This has a negative influence on the intensity of the secondary processes and has long been held as the cause of the presumed low (in comparison with a fluidized bed) thermal efficiency of the vibrating fluidized bed in general and of the high vibrating fluidized bed in particular [2]. On the other hand, if Ω is varied by increasing the height H, the damping of the bed does not change. Consequently, the pressure vibrations p_H are also undamped at the higher-order resonances [32, 39], and this has a positive influence on the hydrodynamics and intensity of the transport processes in the bed [29]. The phase angle φ at resonance, as usual, is $\simeq\pi/2$. On the whole, the experimental points [32, 43] in Fig. 1 exhibit good correspondence with the calculated values for $\Omega < 2$, when the structure of the model system most completely mirrors the true structure. For large values of Ω, quantitative similarity is not always observed (points 2 in Figs. 1b and 1c), because of inhomogeneity and instability of the structure of the higher layers [29].

For $\Theta_v < 0.5$ the resonance fluctuations of the dimensionless gas pressure and the frequencies obtained by the var H and var ω procedures coincide: $\eta_H \simeq \eta_\omega$ and $\Omega_H \simeq \Omega_\omega$ (Fig. 2). Thereafter ($\Theta_v > 0.5$) we have $\eta_H > \eta_\omega$ and $\Omega_H > \Omega_\omega$, where the values of Ω_ω remain constant. The independence of the resonance frequency Ω_ω from the friction forces is one of the fundamental postulates of the classical theory of vibrations [44]. However, its extension to other techniques of "obtaining" the resonance curves [10] is clearly unjustified.

The form of the lower boundary condition has a strong influence on the qualitative and quantitative correspondence with the experimental results for $\Theta_v > 1$. As mentioned, Eq. (8) places all soft (matrix-free) regimes of interaction between the bed and the bottom, including regimes with particle separation from the bottom, within finite limits. Consequently, the agreement of the $\eta_H(\Theta_v)$ curve with the experimental for $\Theta_v > 1$ should not be unexpected. As an illustration, Fig. 2a (curve 14) shows the results of calculations according to the relation

$$\eta^* = \eta(x)\, r_v, \tag{12}$$

which is obtained when Eq. (7) is solved with the usual "nonseparation" condition for homogeneous media: $D_x P(H, t) = \mathrm{Re}[j_V \exp(i\omega t)]$. It is evident that the behavior of Eq. (12) deviates considerably from (11) for $\Theta > 0.5$-1. A similar result is obtained (for the same reasons) from calculations according to a rheological viscoelastic model, for which the corresponding relations are given below (curve 15 in Fig. 2a).

In addition to the resonance frequencies, the system has numerous "optimum" frequencies Ω^p corresponding to the maxima of the dimensioned pressure p as a function of the bed height or the vibration frequency* (Fig. 2c). The values of Ω^p depend on the vibration excitation conditions (i.e., on whether A_V, V_V, or K_V is constant). The curves of $\Omega_H{}^p$ (under all conditions) and $\Omega_\omega{}^p$ (for const V_V) coincide with the Ω_H and Ω_ω resonance curves, because the dimensioned factor P_V, which is used to establish the relation (11) between p and η, does not depend on ω in this case. The situation is otherwise for const A_V and const K_V. Here P_V increases and offsets the above-resonance decrease of η as ω is varied at a constant vibration amplitude, and this causes the extremum of p to vanish at Θ_v = 1.728. Consequently, the $\Omega_\omega{}^p$(const A_V) curve in Fig. 2c is cut off. When the vibrational acceleration is constant (const K_V), the function $P_V(\omega)$ becomes hyperbolic, the resonance curve is smoothed, and the maximum of p shifts into the domain $\Omega_\omega{}^p < 1$ (Fig. 2c).

It follows from [3, 4] that the variation with an increase in the parameter Θ_v according to Eq. (7) is much slower. This is evidenced by data on the pressure mode shapes in the bed (Fig. 3a). For $\Theta_v<1$ the wave profile is nearly sinusoidal. For a higher frequency $\Theta_v\gg1$ the distribution (at $\Omega \simeq 1$) degenerates into a linear dependence, which is a significant departure from the calculations in [3, 4] according to a parabolic equation. Of the latter results, curve 8 in [4] is situated higher, having been obtained under a Neumann-type boundary condition, whereas a Dirichlet-type boundary condition is used in [3]. On the other hand, the solution (11) goes over to an exponential dependence (of the type in [3], curve 7) when the depth of the bed is much greater than the elastic wavelength (curve 9 in Fig. 3b).

The maximum (asymptotic) values of the pressures in the problems [3, 4] exceed the true values by 30-100%, which follows from the principle of equal potential energy of the layers in the variant components. Bearing this fact in mind, we replace the complex analytical relations of [3, 4] by the simple asymptotic expression

$$\eta_{ef} = \sqrt{2\Theta_v}/r_v. \tag{13}$$

This expression gives an estimate of the limiting pressures described by the parabolic equation for the piezoelectric conductivity in the upheaved semiinfinite porous mass when the height H is replaced in the solution of [3] by the effective penetration depth of piezoelectrically generated waves $H_\Phi \sim \sqrt{2a_0^2\tau_v/\omega}$, which is obtained from the solution (7) for the case $\Theta_v \gg 1,\ \Omega \gg 1$.

It is evident from Fig. 2a (curve 16) that for $\Theta_v \geqslant \Theta_v'' = 2$ the function (13) agrees fairly well both with the continuum model and with the experimental data. For $\Theta_v<2$ curve 16 exhibits a characteristic inflection, which mirrors the greater damping of the displacements of the dense layer relative to the hard surface. Since this is the state in which the "continuous" phase usually exists in an inhomogeneous fluidized bed, it is readily perceived that any motion of this phase, including chaotic motion, at a frequency below $\omega \sim 1/\tau_V$ will be damped. This accounts for the strong qualitative similarity between curve 16 and the standard curve of the maximum heat-transfer coefficient in a fluidized bed as a function of the particle diameter [45]. In particular, it follows from this result that the forces of consolidation of small particles in the FS are more of a hydrodynamic than a molecular [45] origin, consistent with the rigorous analysis in [20]. In a vibrating fluidized bed, which is considerably more homogeneous than a plain fluidized bed in the suspended regime and exists in a stable expanded state for $\Theta_v' < \Theta_v < \Theta_v'''$, the vibrations of all scales are of a slightly damped elastic nature,† and the maximum heat-transfer coefficients are close to their limiting values up to micromillimeter particles.

The limit $(\Theta_v<1)$ for application of the rheological analytical model in [9] instead of the continuum model is also obvious from this investigation.

The equation for the forced vibrations of a suspended vibrating bed differ from [9] in the emergence of inhomogeneity on the right-hand side:

$$\ddot{y} + \omega_0^2\tau_v\dot{y} + \omega_0^2 y = A_B\omega^2\cos(\omega t). \tag{14}$$

*Experimental confirmation can be found in [32, 42].

†This contrasts with [10], according to which the damping of vibrations in the vibrating bed, as in a dense layer, begins at a frequency $\omega \sim 1/\tau_V$.

The action of Eq. (14) can be illustrated by representing the FS by a set of particle monolayers separated by thin gas spaces. Motion is imparted to the n-th layer in this case by the elastic and viscous resistance forces:

$$\delta y_n = \frac{P_0}{\varepsilon_0 \rho_b d^2} ((\delta y_{n+1} - 2\delta y_n + \delta y_{n-1}) + \tau_v (\delta \dot{y}_{n+1} - 2\delta \dot{y}_n + \delta \dot{y}_{n-1})). \quad (15)$$

The summation of these resistances and their averaging over the height yields the left-hand side of Eq. (14). Solving it for the steady state and calculating the pressure as $p = \rho_{b0} H \omega_0^2 (y + \tau_V \dot{y})$, we obtain

$$p = \frac{\pi}{2} \frac{P_V \Omega r_v}{\sqrt{(1-\Omega^2)^2 + \Theta_v^2}} \cos(\omega t - \psi); \quad \operatorname{tg} \psi = \frac{\Omega^2 \Theta_v}{r_v^2 - \Omega^2}; \quad (16)$$

$$\eta_H = \frac{\pi}{2} \frac{r_v}{\sqrt{2(r_v - 1)}}; \quad \eta_\omega = \frac{\pi}{2} \frac{\Omega_\omega}{\sqrt{3\Theta_v^2 + 2(1 - \sqrt{r_v^4 + \Theta_v^4})}};$$

$$\Omega_H = \sqrt{r_v}; \quad \Omega_\omega = \sqrt{\sqrt{r_v^4 + \Theta_v^4} - \Theta_v^2}.$$

A comparison of calculations according to Eq. (16) with the preceding analysis indicates (Figs. 1c and 2c, dashed curves) the advantage of using the model (14) in engineering calculations under the stated conditions.

NOTATION

A, amplitude; a, sound velocity; d, particle diameter; F, inertial force; f, frequency; g, gravitational acceleration; H, bed height; h, relative height; j, specific inertial force; K, K_1, K_2, wave number and its real and imaginary parts; L, governing transverse dimension; P, p, pressure (of the gas); R, Reynolds number; r, modulus of a complex number; S, path traveled by the gas in one vibration half-period; t, time; U, u, gas velocity; V, v, particle velocity; x, vertical coordinate; y, height-average deformation of the bed; β, fluid friction coefficient of the fluidized system; δ, effective depth; ε, e, porosity; η, dimensionless pressure (of the gas); Θ_v, dimensionless vibration frequency; λ, wavelength; ν, kinematic viscosity of the gas; ρ, density (of the gas); τ_V, phase velocity relaxation time; φ, ψ, phase angle; Ω, relative frequency; ω, vibration frequency; ω_0, natural frequency of the fluidized system. Indices: v, vibration; h, hydrodynamic; c, convective; m, mass; 0, equilibrium, average, or initial value; b, bed; P, particle; H, ω, varied parameter in the system; p, corresponding maximum of the function p. Other symbols: $D = d/dt$; $D_t = \partial/\partial t$; $D_x = \partial/\partial x$; ∇^2, Laplace operator. Dimensionless and dimensioned groups: $a_0^2 = P_0/(\rho_{b0}\varepsilon_0)$; $h = x/H$; $j_V = \rho_b A_V \omega^2$; $K_V = A_V \omega^2/g$; $R_V = \omega d^2/\nu$; $R_C = (V_V + U_0) d/\nu$; $r_v = \sqrt{1 + \Theta_v^2}$; $V_V = A_V \omega$; $\beta = \frac{150(1-\varepsilon)^2 \rho\nu}{\varepsilon^3 d^2}$; $\Theta_v = \omega\tau_v$; $\rho_b = \rho_p(1 - \varepsilon)$; $\tau_v = \rho_b/\beta$; $\Omega = \omega/\omega_0$; $\omega_0 = \pi a_0/(2H)$.

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LAMINAR FLOW OF A LIQUID IN A ROTATING CYLINDER IN A GRAVITATIONAL FIELD

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The laminar flow of a viscous liquid in a vertical rotating cylinder is studied.

We consider the axisymmetric flow of a viscous incompressible liquid in a semiinfinite rotating circular cylinder, where the axis of the cylinder is in the vertical direction (Fig. 1). The approximate dimensionless system of equations describing the flow of the liquid in cylindrical coordinates, together with the initial and boundary conditions, has the form [1]:

$$rU=\frac{\partial \Pi}{\partial r}-\frac{1}{\mathrm{Re}}\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial rV}{\partial r}, \tag{1}$$

$$r\frac{\partial U}{\partial z}=\frac{1}{\mathrm{Re}}\left[3\frac{\partial U}{\partial r}+r\frac{\partial^2 U}{\partial r^2}-\frac{r}{2U}\left(\frac{\partial U}{\partial r}\right)^2\right], \tag{2}$$

$$\frac{\partial W}{\partial z}=-\frac{\partial \Pi}{\partial z}+\frac{1}{\mathrm{Fp}}+\frac{1}{\mathrm{Re}\,r}\frac{\partial}{\partial r}r\frac{\partial W}{\partial r}, \tag{3}$$

$$\frac{1}{r}\frac{\partial rV}{\partial r}+\frac{\partial W}{\partial z}=0, \tag{4}$$

$$W|_{z=0}=1,\quad V|_{z=0}=0,\quad U|_{z=0}=0,\quad \Pi|_{z=0}=P_0/\rho v_0^2, \tag{5}$$

$$W|_{r=1}=0,\quad V|_{r=1}=0,\quad U|_{r=1}=(\omega_0 R/v_0)^2=\omega^2, \tag{6}$$

where

$$V=\frac{v_r}{v_0};\quad U=\frac{1}{r^2}\left(\frac{v_\varphi}{v_0}\right)^2;\quad W=\frac{v_z}{v_0};$$

$$\Pi=P/\rho v_0^2;\quad \mathrm{Re}=\frac{Rv_0}{\nu};\quad \mathrm{Fp}=\frac{v_0^2}{gR}.$$

The system of equations (1) through (4) is obtained by replacing the derivatives $W\frac{\partial}{\partial z}$ by $\frac{\partial}{\partial z}$ with the assumption that the radial component V of the velocity is small in comparison with the axial W and rotational $\sqrt{r^2U}$ components, and the derivative of the flow in the direction of the axis is much smaller than the derivatives with respect to the radial coordinate [2].

According to the method given in [1] for solving the system (1) through (6), we look for the function Π in the form:

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